EXERCISES WEEK 39

1) Exercise 2.2 a-d from K&R

2)
The momentum equations for ions and electrons are given by

$$m_i \frac{d\vec{v}_i}{dt} = e\left(\vec{E} + \vec{v}_i \times \vec{B}\right) \qquad \text{Eq. 1.1}$$

$$m_e \frac{d\vec{v}_e}{dt} = -e\left(\vec{E} + \vec{v}_e \times \vec{B}\right) \qquad \text{Eq. 1.2}$$

a) Assume a static uniform electric field along the y-axis and a static uniform magnetic field along the *z*-axis. Sketch the particle trajectories separately for electrons and ions.

b) Show that the zeroth order drift of the guiding center is given by

$$\vec{v}_{gc} = \vec{u}_E = \frac{\vec{E}_\perp \times \vec{B}}{B^2}, \qquad \text{Eq. 1.3}$$

independent of both the mass and charge.
{ *Hint:* $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ }

c) Assume a magnetic field along the *z*-direction increasing in strength with increasing *y*, and no electric field. Draw a sketch showing the particle trajectories separately for ions and electrons.

Assume that the magnetic field strength increase linearly with increasing *y*.

d) Show that the magnetic field can be expressed as

$$B_z = B_{0z} + \left(\frac{\partial B_z}{\partial y}\right) r_c \cos\phi \qquad \text{Eq. 1.4}$$

where $r_c = \frac{mv_\perp}{eB_z}$ is the gyro radius, and ϕ is the angle that the position vector $\vec{r} = (x,y)$ makes with *y* in a guiding center reference frame.

e) Show that the average forces over one gyro-period are given by

$$\langle F_x \rangle = \frac{-qv_\perp}{2\pi} \int_0^{2\pi} \left(B_{0z} \sin\phi + r_c \left(\frac{\partial B_z}{\partial y}\right) \sin\phi \cos\phi \right) d\phi \qquad \text{Eq. 1.5}$$

and

$$\langle F_y \rangle = \frac{-qv_\perp}{2\pi} \int_0^{2\pi} \left(B_{0z} \cos\phi + r_c \left(\frac{\partial B_z}{\partial y}\right) \cos^2\phi \right) d\phi \qquad \text{Eq. 1.6}$$

in *x* and *y* directions, respectively.

f) Integrate Eqs. 1.5 and 1.6 and derive the following expression for the gradient drift of the guiding center:

$$\vec{v}_{gc} = -\frac{1}{2} \frac{mv_\perp^2}{qB_z^2} \left(\frac{\partial B_z}{\partial y}\right) \hat{x} \qquad \text{Eq. 1.7}$$

{*Hint:* $\int \cos^2 x \, dx = \frac{1}{2}\sin x \cos x + \frac{1}{2} x + C$; $\int \sin x \cos x \, dx = \frac{1}{2}\sin^2 x + C$ }

The general expression for the gradient drift is:

$$\vec{u}_{\nabla B} = \frac{1}{2} mv_\perp^2 \frac{\vec{B} \times \nabla \vec{B}}{qB^3} \qquad \text{Eq. 1.8}$$

3) Exercise 2.4 from K&R